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From (4) we find $x^2y^2 = x^4 - \dfrac{x^4z^2}{4(r^2 - x^2)} = \text{a maximum}. \;.\;.\;.\;.\; (5)$

Differentiating (5), regarding z as constant, equating to zero, and reducing, we have $(2r^2 - x^2)z^2 = 8(r^2 - x^2)^2. \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. (6)$

From (6) we have $z^2 = \dfrac{8(r^2 - x^2)^2}{2r^2 - x^2}. \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (7)$

Substituting the value of z^2 in (5), reducing, and extracting the square root, we have $xy = \dfrac{x^3}{(2r^2 - x^2)^{\frac{1}{2}}}. \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. (8)$

Differentiating (7), we have $zdz = \dfrac{8r^4xdx}{(2r^2 - x^2)^2} - 8xdx. \;.\;.\;.\;.\;.\;.\;.\;.\; (9)$

If A denotes the average area of the ellipse, we have

$$A = \frac{\int_0^{2r} \pi xy.2\pi zdz}{\int_0^{2r} 2\pi zdz} = \frac{\pi}{2r^2}\int_0^{2r} xyzdz. \;.\;.\;.\;.\;.\; (10)$$

Substituting in (10) from (8) and (9), and observing that when $z = 0$, $x = r$, and when $z = 2r$, $x = 0$, we have

$$A = \frac{4\pi}{r^2}\int_0^r \frac{x^4dx}{(2r^2 - x^2)^{\frac{1}{2}}} - 4\pi r^2\int_0^r \frac{x^4dx}{(2r^2 - x^2)^{\frac{5}{2}}}. \;.\;.\;.\;.\;. (11)$$

Putting $\sin\theta = x \div r\sqrt{2}$, (11) becomes

$$A = 16\pi r^2\int_0^{\frac{1}{4}\pi} \sin^4\theta d\theta - 4\pi r^2\int_0^{\frac{1}{4}\pi} \tan^4\theta d\theta = \pi r^2\left(\frac{\pi}{2} - \frac{4}{3}\right).$$

[PROB. 89. Though we have already published two solutions of this problem, yet as both are in some respects confessedly defective, and as the method adopted in the following solution differs from both the published solutions, we add another solution of 89.

In justice to Mr. Siverly and to Mr. Adcock, we take pleasure in stating that Mr. Siverly corrected his statement that the spheres would roll in opposite directions, several months ago, and by rigorous analysis had arrived at the same conclusion, in relation to the rolling of the lower sphere, as that deduced in the following solution by Mr. Hill. And, since the issue of No. 2, we have received from Mr. Adcock a revision of his published solution, in which, after correcting equations (6) and (9), he obtains an equation for the required curve which corresponds with that given by Mr. Siverly, and consequently with the following by Mr. Hill. And he also arrives at the same conclusion as Mr. Siverly and Mr. Hill, in relation to the rolling of the lower sphere.